ROUGH ESTIMATE OF THE HEAT TRANSFER COEFFICIENT IN LAMINAR AND IN TURBULENT FLOW THROUGH FLAT CHANNELS

Yu. A. Gavrilov and G. N. Dul'nev

UDC 536.242

Approximate formulas are proposed for estimating the coefficient of convective heat transfer in laminar and in turbulent flow at the entrance to a flat channel.

We consider the convective heat transfer during the flow of fluid through the entrance segment of a flat channel, where both the velocity profile and the temperature profile originate. We will make the following assumptions usually stipulated in such problems [1]: the flow and the heat transfer processes are steady, the physical properties of the fluid are constant, the temperature and the velocity at the channel entrance ($x = 0$) are uniform over the cross section, a constant thermal flux density is maintained at the inside wall surface, energy losses and heat conduction in the axial direction are negligible.

A similar problem of laminar flow was solved by Siegel and Sparrow [2] and was considered in [1, 3]. An exact solution for the initial hydrodynamic and thermal stages of a channel was presented in a rather unwieldy form very difficult to use in engineering practice. With an insignificantly larger error, however, it is possible to derive sufficiently simple relations.

It has been shown in [1, 2] that at $x \geq l_i$ the Nusselt number is constant and equal to

$$\mathrm{Nu}_\infty = \frac{\alpha h}{\lambda} = 4.12, \tag{1}$$

while at sufficiently short distances x the equation of local heat transfer for a stream along an immersed plate

$$\mathrm{Nu} = 0.46\left(\frac{h\,\mathrm{Re}}{x}\right)^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{3}} \tag{2}$$

is applicable. We will assume, furthermore, that Eq. (2) is valid for the entire initial stage. We consider a fluid with the Prandtl number $\mathrm{Pr} \approx 1$, where both the hydrodynamic and the thermal initial stages are of equal length. That length is determined, according to [1], from the equality

$$l_i = a_1 h \mathrm{Re}. \tag{3}$$

Factor a_1 is assumed constant. Its value depends on the conditions under which the edges of the boundary layer and the coordinates of their termination in the channel are estimated. At $x < l_i$, the velocity and the temperature of the fluid in the outer edge region of the boundary layer approach asymptotically their mainstream values as that edge moves closer to the axis. At $x \approx l_i$ the thickness of the boundary layer approaches 0.5 h, also asymptotically, with increasing x. For this reason, in rough calculations we allow some leeway in defining a_1. We will determine a_1 from (2) and (3), letting $x = l_i$ and $\mathrm{Nu} = 4.12$:

$$a_1 = 0.0125\mathrm{Pr}^{\frac{2}{3}}. \tag{4}$$

Then, taking into account (4), we insert into (2) the value of hRe from (3):

Institute of Precision Mechanics and Optics, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 23, No. 4, pp. 612-617, October, 1972. Original article submitted April 26, 1972.

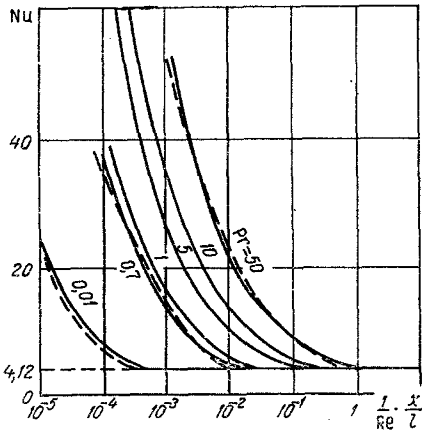

Fig. 1

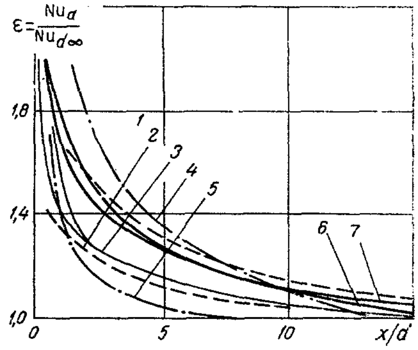

Fig. 2

Fig. 1. Heat transfer coefficient in laminar flow: solid lines represent exact solution; dashed lines represent approximate solution.

Fig. 2. Comparison between approximate and experimental relations $\varepsilon = \varepsilon(x)$ for turbulent flow: 1) upper dashed line according to [13] with $Re_d = 7.1 \cdot 10^3$; 2) [13], $Re_d = 7.2 \cdot 10^5$; 3) [15]; 4) [11], $\bar{K} = 20\%$; 5) [11], $\bar{K} = 0$; 6) formula (15); 7) formula (16).

TABLE 1. Values of l_i and n in (8) for Laminar Flow

Pr	l_i	n
0,6>Pr	0,029 hPe	0,5
0,6⩽Pr⩽10	0,0125 $\Pr^{\frac{2}{3}}$ hRe	0,5
Pr>10	0,0243 $\Pr^{\frac{5}{6}}$ hRe	0,4

$$\mathrm{Nu} = 4.12\left(\frac{l_i}{x}\right)^{0.5}. \tag{5}$$

The value of $\overline{\mathrm{Nu}}$ at $0 < x \le l_i$ will be determined from the relation

$$\frac{1}{\overline{\mathrm{Nu}}} = \frac{1}{x}\int_0^x \frac{dx}{\mathrm{Nu}}, \tag{6}$$

and at $x \ge l_i$ according to the formula

$$\overline{\mathrm{Nu}} = \frac{l_i \overline{\mathrm{Nu}}_1 + (x - l_i)\mathrm{Nu}_\infty}{x}. \tag{7}$$

On the right-hand side of (7) $\mathrm{Nu}_\infty = 4.12$ and $\overline{\mathrm{Nu}}_1$ is taken from (6) at $x = l_i$. We finally have

$$\begin{gathered}\mathrm{Nu} = \left(\frac{l_i}{x}\right)^n \mathrm{Nu}_\infty;\ \overline{\mathrm{Nu}} = (1+n)\left(\frac{l_i}{x}\right)^n \mathrm{Nu}_\infty \quad \text{at} \quad x \leqslant l_i,\\ \mathrm{Nu} = \mathrm{Nu}_\infty;\ \overline{\mathrm{Nu}} = \left(1 + n\frac{l_i}{x}\right)\mathrm{Nu}_\infty \quad \text{at} \quad x \geqslant l_i.\end{gathered} \tag{8}$$

Let us compare this answer with the exact solution. The relation Nu = f(x), which has been based in [1] on the exact solution, is shown in Fig. 1 by solid lines. On the diagram is also shown a dashed line representing Eq. (8) for Pr = 0.7. Evidently, the approximate solution (8) agrees fairly well with the exact one.

For a Prandtl number very different from unity, it is suggested that the values of n and l_i in Eqs. (8) be taken from Table 1. The dashed lines in Fig. 1, calculated for Pr = 0.01 and Pr = 50, are shown here for comparison. The agreement is satisfactory. We note that in [4] are also given various formulas for calculating the Nusselt number for a laminar flow along a plate, as a function of the limits of possible Prandtl number values given in Table 1.

We will now proceed to analyze the heat transfer in a turbulent stream. In this case, a single formula is given in [4] for a stream along a plate with any value of the Prandtl number. The heat transfer in the stabilized channel segment, however, depends on the Prandtl number. We will analyze the heat transfer for the case of a liquid and gases only. For the segment of stable heat transfer we use the formula in [5]:

$$\mathrm{Nu}_{d\infty} = \frac{\alpha d}{\lambda} = 0.022\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^{0.43}. \tag{9}$$

Formula (9) has been obtained for channels with circular sections, but is applicable also to flat and rectangular channels [6-8]. For a flat channel one may, therefore, use the results of heat transfer studies made on a cylindrical tube. It is also obvious that the heat transfer in a turbulent stream depends very little on the boundary conditions at the wall surfaces [8-10]. In the channel entrance segment, however, the heat transfer depends additionally on the initial stream turbulization and on the shape of the entrance edge [11-14].

Most studies concerning the heat transfer in tubes (for example, [12, 13]) were made with a stable velocity profile at the entrance to the heated segment. In [13] the heat transfer in a cylindrical tube was studied so as to take into account the longitudinal flow of heat along the walls and through the liquid. It has been shown there that the segment of thermal stabilization is not over 30 diameters long and may, within 5% accuracy, be confined to a distance up to 16 diameters long. The values obtained for ε, i.e., the ratio $Nu_d/Nu_{d\infty}$ of the local Nusselt number to the Nusselt number at infinity are shown in Fig. 2 by dashed lines for Pr = 0.8. Curve 1 corresponds to $Re_d = 7.1 \cdot 10^3$ and curve 2 corresponds to $Re_d = 7.2 \cdot 10^5$. The curves corresponding to intermediate values of Re_d will lie between these two. Within the same range we find the curves for Pr = 0.71 and $Re_d = 3 \cdot 10^3$-$5 \cdot 10^4$ [12] corresponding to heat transfer with a constant thermal flux at the walls. It has been asserted in [15] that the heat transfer process stabilizes over a distance $l_i = 16.5d$, and values for ε have been obtained there which are shown in Fig. 2 by the solid line 3. This line splits at $x < 1.5d$ into a family of curves representing different values of the Reynolds number. The upper branch corresponds to $Re_d = 5 \cdot 10^3$, the lower branch corresponds to $Re = 10^5$. Curve 3 with an upper branch approximately fits the equation

$$\varepsilon = 1.38 \left(\frac{x}{d}\right)^{-0.12} \tag{10}$$

proposed in [5].

The effect of initial stream turbulization was studied in [11], where the following relations were suggested:

$$\varepsilon = A \left(\frac{x}{d}\right)^{-m}; \; l_i = dA^{\frac{1}{m}}, \tag{11}$$

with $A = 1.35 + 0.04\bar{K}$, $m = 0.17 + 0.006\bar{K}$, and $K = \Delta u/u$ (%).

The dashed-dotted curves 4 (for $\bar{K} = 20\%$) and 5 (for $\bar{K} = 0$) in Fig. 2 have been plotted according to Eq. (11). The initial stream turbulization, estimated in terms of the Karman number $\bar{K}$ (mean-over-the-section ratio of the mean-squared pulsations of axial velocity Δu to the mean stream velocity u), is usually not known in the solution of application problems.

It follows from Fig. 2 that the various $\varepsilon = \varepsilon(x/d; Re)$ curves obtained by different authors do similarly reflect the trend of the relation between ε and Re. In many cases the $\varepsilon = \varepsilon(Re)$ curves cannot be compared, inasmuch as they have been obtained for different test conditions (shape of the inlet edge, initial turbulization, etc.). Besides, there are not sufficient test data available for making correct generalizations. We analyze the relation $\varepsilon = f(x/d)$ in the first approximation only, therefore, and average out the effect of other parameters.

We assume that the heat transfer coefficients for the channel entrance segment can be estimated according to the formula analogous to that for a turbulent stream along a flat plate [4]:

$$Nu_x = a_2 Re_x^{0.8} . \tag{12}$$

Multiplying both sides of this equality by d/x, we have

$$Nu_d = a_2 \left(\frac{d}{x}\right)^{0,2} Re_d^{0,8} . \tag{13}$$

The constant coefficient a_2 will be determined from the condition that at $x = l_i = 16d$ the right-hand sides of (9) and (13) become identical:

$$a_2 = 0{,}038 Pr^{0.43}. \tag{14}$$

Dividing (13) by (9), with (14) taken into account, we have for $x \leq 16d$

$$\varepsilon = 1.74 \left(\frac{x}{d}\right)^{-0.2}. \tag{15}$$

Equation (15) is represented by curve 6 in Fig. 2. We will use the average value of the power exponent in (10) and (15), making it equal to −1/6, and will write

$$\varepsilon = 1.65\left(\frac{d}{x}\right)^{\frac{1}{6}} \quad \text{at} \quad x \leqslant 20d. \tag{16}$$

The factor 1.65 in (16) has been obtained from the condition $\varepsilon = 1$ at $x = 20d$. Curve 7 has been plotted according to (16) and is a more convenient average for the values obtained in these studies of heat transfer in channels.

In order to determine the mean value of $\overline{Nu}_d$, we will use relations of the (6) and (7) kind. Then, after necessary calculations and transformations, we arrive at formula (8). For a turbulent stream, however, the governing dimension in (8) is the equivalent diameter, with $l_i = 20d$ and $n = 1/6$. The approximate formulas (8) are relatively simple and may be recommended for practical calculations.

NOTATION

x is the longitudinal coordinate;
l_i is the length of the initial segment;
α is the coefficient of convective heat transfer;
λ is the thermal conductivity of the fluid;
h is the distance between the channel walls;
d is the equivalent hydraulic diameter;
Nu is the Nusselt number;
Re is the Reynolds number;
Pe is the Peclet number;
Pr is the Prandtl number;
$\overline{K}$ is the Karman number;
$\varepsilon = Nu/Nu_\infty$.

Subscripts

∞ denotes beyond initial segment;
d, x denote governing dimensions;
bar above a symbol denotes the average value.

LITERATURE CITED

1. B. S. Petukhov, Heat Transfer and Drag in Laminar Flow through Pipes [in Russian], Énergiya (1967).
2. R. Siegel and E. M. Sparrow, AIChE J., 5, No. 1 (1959).
3. Mercer, Pierce, and Hitchcock, Heat Transmission, No. 3 (1967).
4. G. Schlichting, Theory of the Boundary Layer [Russian translation], Nauka (1969).
5. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Énergiya (1969).
6. N. I. Buleev, V. A. Mosolova, and L. D. El'tsova, Teplofiz. Vys. Temp., 5, No. 4 (1967).
7. N. S. Alferov and R. A. Rybin, Convective Heat Transmission in Two-Phase and in One-Phase Streams [in Russian], Énergiya (1964).
8. V. M. Keyes and A. L. London, Compact Heat Exchangers [Russian translation], Énergiya (1967).
9. Lecuyer and Warner, Heat Transmission, No. 1 (1962).
10. A. I. Leont'ev, B. P. Mironov, and V. A. Mukhov, Inzh.-Fiz. Zh., 15, No. 1 (1968).
11. E. P. Dyban and É. Ya. Épik, ibid., 14, No. 4 (1968).
12. Reynolds, Svirenten, and MacElliot, Theoretical Principles of Engineering Calculations [Russian translation], No. 1, Mir (1969).
13. N. I. Buleev, L. D. El'tsova, and G. P. Biryukova, Teplofiz. Vys. Temp., 4, No. 4 (1966).
14. S. S. Filimonov and B. A. Khrustalev, Heat and Mass Transfer [in Russian], Gosénergoizdat (1963).
15. V. M. Legkii and A. S. Makarov, Inzh.-Fiz. Zh., 20, No. 2 (1971).